Project A

Mathematics: ★★★★☆ Creativity: ★★★★☆ Impact: ★★★★★ Programming: ★★★★☆

Master/BEP Project in the Geometric Learning and Differential Geometry group (CASA), link.

Tracking of Vascular Trees with Vessel Boundaries and Artery-Vein Classification via Equivariant Deep Learning in Orientation Scores.

Background and problem formulation: Automatic Tracking of the vascular tree in retinal images is important for screening and early diagnosis of diseases such as diabetes, Alzheimer's disease and glaucoma. We closely collaborate with University of Maastricht (ophthalmology department, Tos Berendschot) on this application. We have achieved full vascular tree trackings link, with recent major breakthroughs in link.

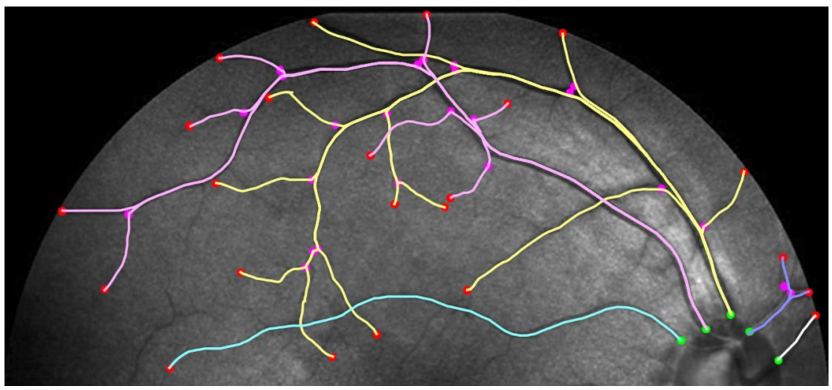


Figure 1a. complete vascular tree tracking link.

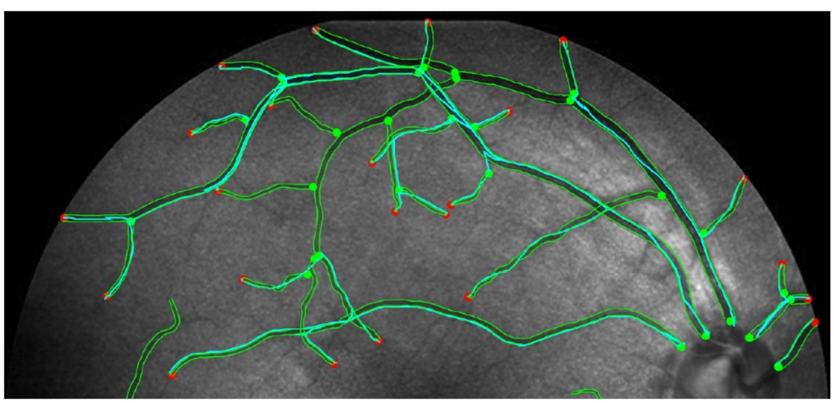


Figure 1b. Full vascular tree tracking with vessel boundaries/widths.

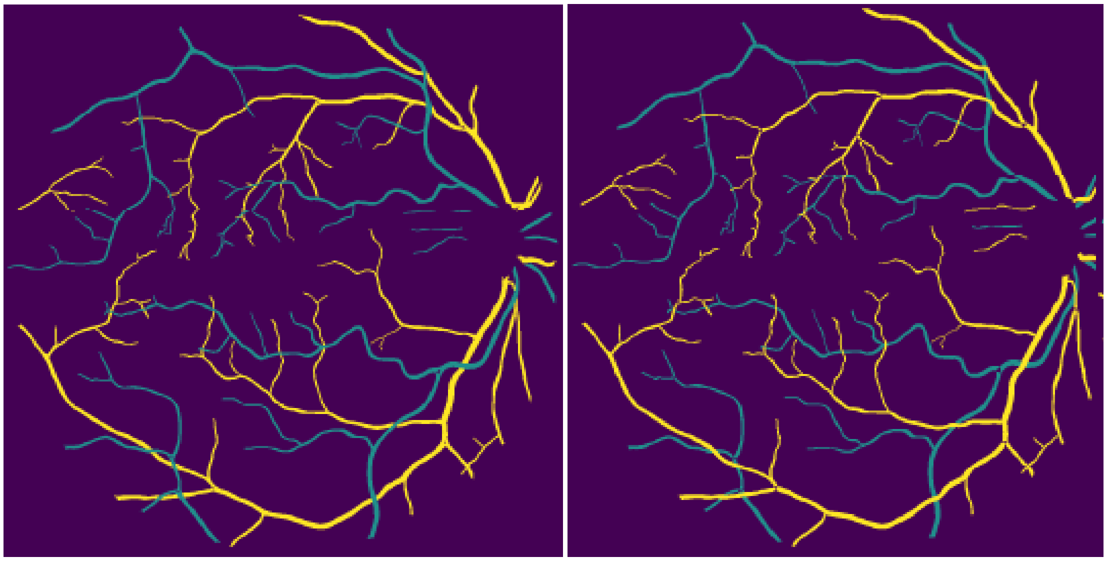


Figure 1c. artery vein segmentations (left: our output of quick test, right: ground truth)

2 things are still missing in our vascular tree tracking (Fig. 1a) for which we have very well-documented Mathematica code see link:

1. Besides extraction of vessel centerlines via geodesic models we need extraction of vessel boundaries. First investigations (Fig. 1b) show that is again possible with geodesic models.
2. We need classifications of arteries and veins, for that we can use PDE-G-CNNs link and morphological components in the space of positions and orientations. First investigations show that this possible (Fig. 1c), but further analysis, development and improvement (as there are still some cases with too many failures) is required.

Expected Output: Report + Presentation+ well-documented Code.

Requirements. This project requires background in PDEs and scientific computing. (It is highly beneficial if the student follows the course Differential Geometry for Image Processing 2MMA70 CANVAS).

For more information on the underlying mathematics:

contact Remco Duits R.Duits@tue.nl and/or Gautam Pai: G.Pai@tue.nl

Daily supervisors:

Nicky van den Berg and Gautam Pai.

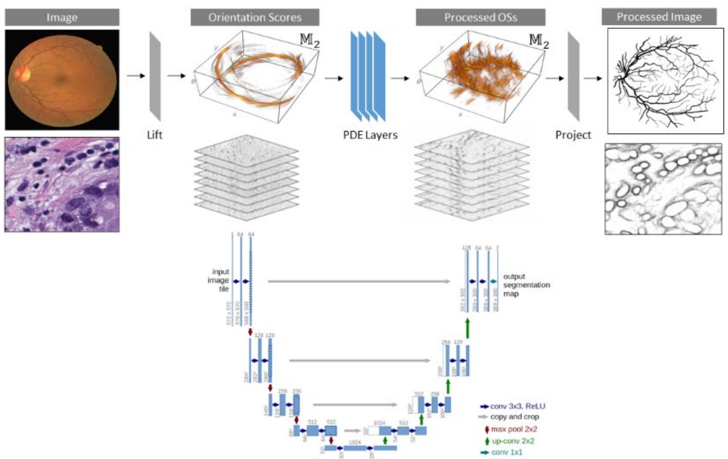
Monthly supervision: Remco Duits and Tos Berendschot (UM).

Project B

Master project in the Geometric Learning and Differential Geometry Group (CASA)

Mathematics: ★★★☆☆ Creativity: ★★★★☆ Impact: ★★★★☆ Programming: ★★★★★

Exploring Advanced Architectures using PDE-based Group Convolutional Neural Networks



Background and problem formulation:

U-Nets (https://lmb.informatik.uni-freiburg.de/people/ronneber/u-net/) are prominent deep-learning networks known to achieve excellent results on biomedical image processing tasks. The vanilla architecture comprises of a U-like shape where an input image is first sequentially convolved and downsampled (maxpooled) and then correspondingly upsampled and convolved back to its original dimension. The key components include convolutions, subsampling/pooling and regular skip connections to ensure efficient integration of low and high-frequency structures. However, just like traditional CNNs are **not** equivariant to rotations and reflections, a classical U-Net also has the same drawback. A preliminary review of existing literature suggests that only a few attempts have been made to make such architectures equivariant. In addition, a comprehensive evaluation of the benefits of doing so is largely unknown.

In this project, we aim to design an equivariant U-Net-like architecture with the conceptual components drawn from the PDE-G-CNN framework (https://bmnsmets.com/publication/smets2022pde/). Thus we will explore a new architecture that can be obtained by replacing widely used operations like convolutions and max pooling in R2 with more sophisticated components like morphological convolutions, erosions, and dilations in a lifted space M2. The seeming advantages of such an approach seem to be (1.) Enforcing equivariance with an architectural construction (2.) Possibly lower memory complexity due to subsampling layers (3.) Hopefully a better performance. In addition, there will be an analysis of how our novel architecture behaves in low data regimes where the availability of good quality training data is a significant constraint.

On the applied side we work on cell boundary-segmentation and mitosis detection in histopathology on which we collaborate with Mitko Veta at BMT. A PDE-G-CNN follow up of link is anticipated.

Information: contact Gautam Pai g.pai@tue.nl

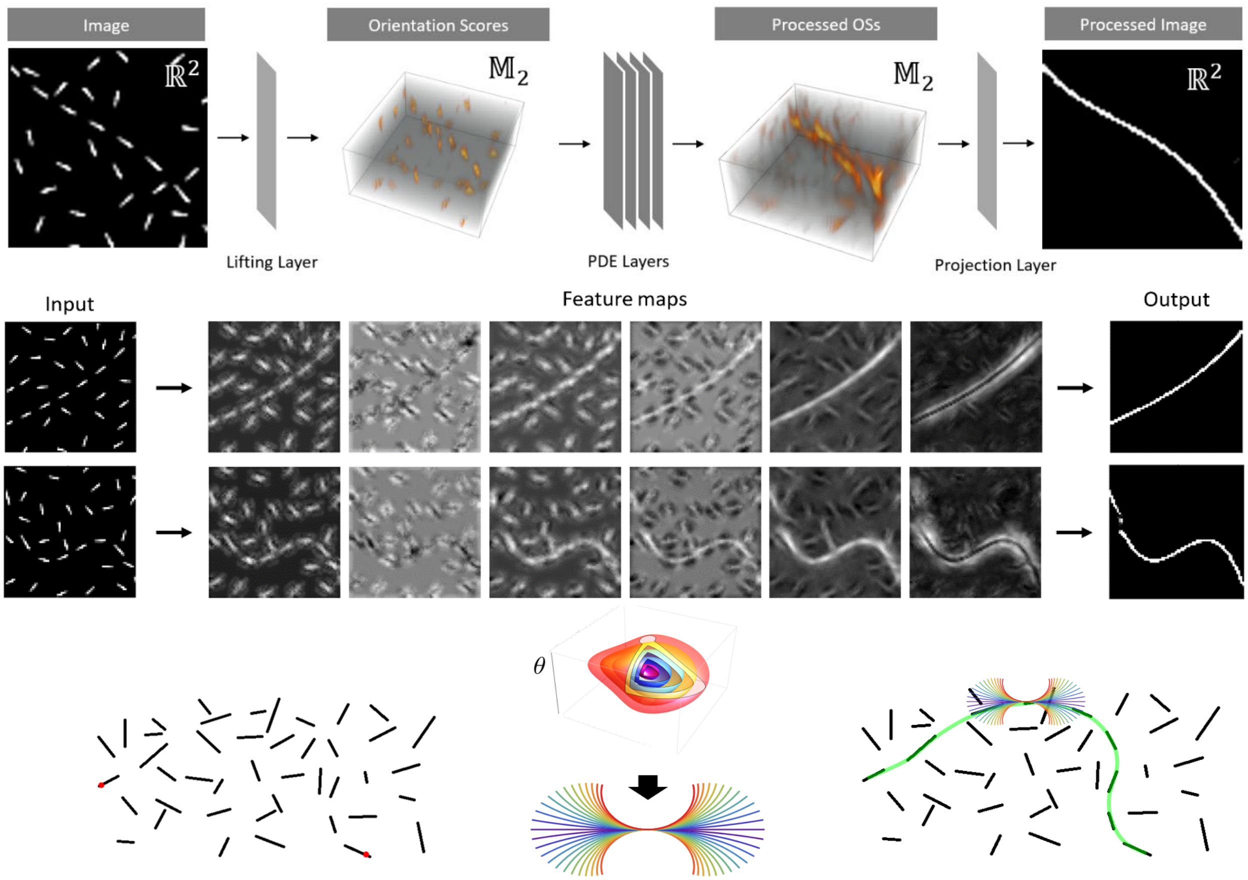
Daily supervisor: G. Pai. Co-supervisors: B.M.N. Smets, M.Veta (BMT) and R.Duits.

Project C

Axiomatic Theoretical Underpinning and Testing of Equivariant PDE-Based CNNs for Line Perception.

Mathematics: ★★★★★ Creativity: ★★★★☆ Impact: ★★★★☆ Programming: ★★★☆☆

Project (Master/Internship) in the [Geometric Learning and Differential Geometry Group]



Background and problem formulation:

The theoretical part of this project will require an axiomatic setup of Equivariant Deep Learning via PDEs on the [Lie group of two dimensional roto-translations] $SE(2)$. The practical part of this project will be mainly focused on experimenting with the [LieTorch] package. LieTorch was made bij Bart Smets and is a publicly available implementation of a [PDE Group Equivariant Convolution Neural Network] (PDE-G-CNN). PDE-G-CNNs are named so because their internals are based on the evolution of certain PDEs. In practice these PDEs are solved approximately with a so-called approximative kernel. The performance of PDE-G-CNNs (might) depend on the choice of approximative kernel, and this is exactly what will be experimented on. Central Questions:

- We now choose certain PDEs for PDE-G-CNNs, can we derive them from axioms and symmetries, likewise the axioms of scale space theory.
- Are the kernels that approximate the PDEs better also the ones that perform better in
  a) Line-recognition tasks (where we currently get dice performance scores of 97%),
  b) Vessel segmentation tasks,
  Keeping in mind stability under small deformations and noise?
- Are the sub-Riemannian models that describe contour perception in human vision also the best for contour perception in artificial vision?

Expected Output:

Report + Presentation + Well-documented code

Daily supervisors:

Gijs Bellaard, Gautam Pai, Remco Duits

For More Information:

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Project D

Equivariant Convolutional Networks for solving the Windkessel model

Mathematics: ★★★☆☆ Creativity: ★★★★★ Impact: ★★★☆☆ Programming: ★★★★☆

Background

Currently, not even half of pre-eclamptic patients is screened positive before symptoms arise link. When screened in time, preventive medicines are available link. Thus screening with a good predictor is important!

A good predictor for pre-eclampsia is the pressure in and with that the resistance of the placental vessels. However, these parameters aren't easily measured by doctors. Therefore, we are developing ML models that couples the flow – which can be measured – to the pressure, resistance and compliances of a vessel. We have modelled a PINNs (physics-informed neural network) that can find this relation. Mathematically seen, this method could be optimised for this problem by taking into account the symmetries in the corresponding PDE (a 3-element Windkessel). These symmetries could be added to the structure of the NN to improve the results and improve the computational time by creating an PDE-G-CNN link. Some symmetries found in the PDE are: translation, scaling.

Project Formulation

Your exercise will be to rewrite this PINN for solving the 3-element Windkessel (WK) model to eventually a PDE-G-CNN. For that, we present you the following roadmap:

1. Identify symmetries in the PDE
2. Write a CPINN (CNN + PINN)
3. Rewrite the standard ReLu convolutions to morphological convolutions
4. Create a G-CNN+ (a Group-CNN incl. the symmetries with morphological convolutions)
5. If time remains; Rewrite to an PDE-G-CNN
    a. *Advantage of this PDE-G-NN is that less parameters are used in the network and with that the training is faster. Also adding these PDEs to the system makes the network more explainable.*

We can provide you with labelled syntethic data. That means, we can generate corresponding flow, pressure and WK-parameters for you.

Challenges

This project requires creativity and application-oriented mindset and has some challenges:

- Both the pressure as well as the WK-parameters should be an output of the network, but not part of the network. The latter since the model should be patient generic but specific, meaning that we cannot train the network for every patient again, since no real data is available. Some symmetries are only between pressure and flow, but some might involve the WK-parameters.

Expected Output:

Report + Presentation + Well-documented code

Daily supervisors:

Pascalle Wijntjes, Wouter Huberts (BME, both with math background),
Bart Smets & Remco Duits (W&I)

For More Information:

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Project E

BEP Project in the Geometric Learning and Differential Geometry Group (CASA)

Alternative Semirings in Neural Networks

Mathematics: ★★★☆☆

Creativity: ★★★★☆

Impact: ★★★★☆

Programming: ★★★★★

Background and problem formulation:

The use of the linear semiring $(\mathbb{R}, +, \cdot)$ is ubiquitous in neural networks as the standard way of combining inputs into an output. There are however alternatives, such as the tropical semiring $(\mathbb{R} \cup \{-\infty\},\ max,\ +)$ and many others. In this project you will explore the use of some alternative semirings in neural networks.

The main challenge will be implementing these semirings as (differentiable) operators in an existing deep learning framework (PyTorch, Flux.jl, ...) and designing and performing various experiments to gauge the impact of the change. This requires strong programming skills.

Information: contact Bart Smets B.M.N.Smets@tue.nl

Daily supervisor: B.M.N. Smets. Monthly supervisor: R. Duits.