Proposal – Joint Master's Project: TU/e–UU

Mathematical Foundations of Inverse Gradient-Index Lens Design

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Introduction

In recent times, the landscape of optical design has undergone a transformative shift with the advent of additive manufacturing techniques, particularly in the realm of gradient-index (GRIN) lenses. Traditionally confined to reflecting and refracting surfaces, optical design has expanded to encompass volumetric variations in refractive index (RI) through GRIN lenses. We refer to the YouTube video (https://www.youtube.com/watch?v=wk67eGXtbIw&t=213s) for the potential of GRIN lenses, to Figure 1 for an illustrative example, and [2] for an explanatory article on the topic. This groundbreaking development offers unprecedented opportunities for optical innovation with implications spanning imaging and non-imaging optics. Applications such as scanning lenses, scopes (endoscope, telescope, borescope), fiber optics, and lithography machines stand to benefit from the simplicity and versatility these lenses promise.

While the inverse problems associated with optical surface design have been addressed using an optimal transport approach [4], determining the RI field for GRIN lenses poses a distinct challenge. If f and g represent source and target distributions of light in X and Y domains respectively, see Figure 1, then the mapping $\boldsymbol{m} : X \to Y$ turning the source into the target needs to satisfy energy conservation

$$g(\boldsymbol{m})\det(D\boldsymbol{m}) = f. \tag{1}$$

However, the mapping must also result from light refracting through the GRIN lens, which restricts how the RI fields change, thus, imposing a closure relation for the system. The closure relations are well-known for reflective/refractive surfaces (e.g., for one mirror system, $\boldsymbol{m}$ has to be the gradient of the mirror height), but not yet known for the GRIN lenses. This master thesis project aims to bridge this gap by leveraging the collective expertise of the Illumination Optics group (https://www.win.tue.nl/~martijna/Optics/) at TU/e and UU, specializing in freeform optics [6, 8] and inverse methods [7, 3].

*The **central goal** is to establish the mathematical foundations for inverse GRIN design, focusing on achieving target light distributions from given source distributions.*

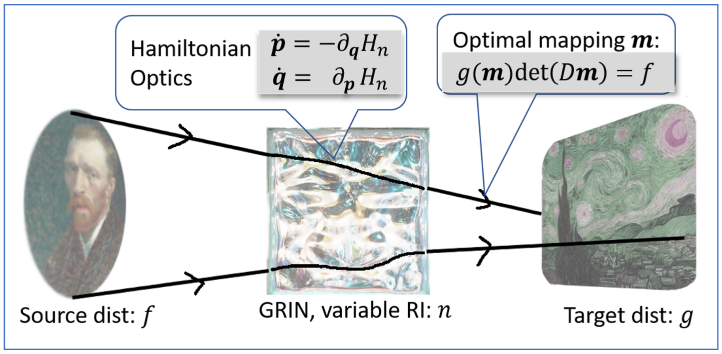

Figure 1. Basic principle of GRIN lens

This project holds significant promise for industrial applications and comes under the umbrella of a larger consortium involving industrial partners such as ASML, Signify, Canon, TNO, along with many others.

Objectives

The objectives can be sub-divided into:

1. **Inverse formulation:** Derive the mathematical formulation of the inverse problem for GRIN design, for example, a closure relation to (1). Alternatively, light can be modeled as a wave due to the Huygens principle, making it possible to apply phase retrieval techniques [1, 5].
2. **Numerical methods:** Designing numerical methods to solve the corresponding inverse problem.
3. **Scientific computing:** Developing rudimentary codes to obtain the RI fields using the method.

The details will be based on the student's preferences/expertise. *Students can hope to gain experience in modelling and scientific computing from the project, work on an application-oriented topic, and build possible connections with industry.*

Pre-requisites: Basic knowledge of analysis, numerical methods, & physics.

References

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