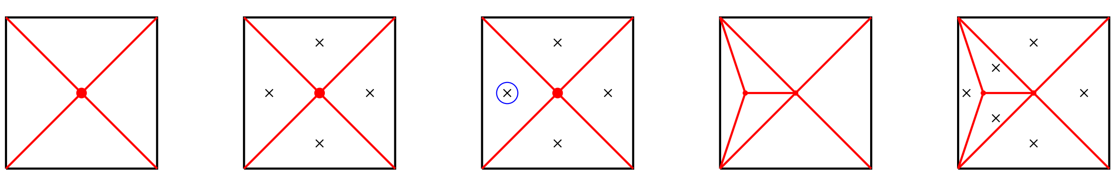
HYPERREDUCTION FOR POLYTOPE DIVISION METHOD

1. Background

Many natural and physical phenomena can be described by partial differential equations (PDEs), which often depend on parameters $\mu \in \mathbb{R}^d$. For a given parameter $\mu \in \mathbb{R}^d$, the solution to the PDE can be approximated using standard numerical methods such as the finite element method. However, in practical applications, the value of the parameter, μ^*, that best characterizes the system may be unknown or subject to change. To determine or approximate μ^*, one then needs to solve an inverse or optimization problem, which in turn typically requires repeated solution of the PDE for many values of the parameter. This is generally very expensive due to the high computational cost of a single PDE solve.

To reduce the computational expense, model order reduction (MOR) techniques could be employed which aim to approximate the solution space of a PDE efficiently, i.e., at low computational cost. One class of MOR techniques is the reduced basis method . A reduced basis is a linear space that approximates the solution manifold defined as $\mathcal{M} \coloneqq \left\{ u(\mu) \middle| \mu \in \mathcal{P} \subset \mathbb{R}^d \right\}$, where $\mathcal{P}$ is the parameter space. A reduced basis has the form $\operatorname{span}\{u(\mu_1), \dots, u(\mu_n)\}$, where $u(\mu_i)$ is the finite element solution for a specific parameter $\mu_i \in \mathcal{P}$. Constructing a linear reduced basis can be advantageous because we want the rank of the span to be low while still constructing a linear space that is a good approximation of the solution manifold. Various algorithms exist to construct a reduced basis, but many algorithms suffer from the curse of dimensionality, meaning that the computational complexity scales exponentially as the dimensional of the parameter space increases. Additionally, the size of the reduced basis can become prohibitively large, resulting in slower computations.

An attempt to overcome the curse of dimensionality is the Polytope Division Method (PDM). PDM is a novel method to determine a reduced basis. The benefit of PDM is that for a fixed rank of the span, PDM scales polynomially to the parameter dimension, unlike most alternative methods. Therefore, PDM is particularly suited for problems with high-dimensional parameter spaces. A sketch of PDM is given by Figure 1.



Step 1: Sampling of the first parameter fig: sf1: PDM2d

Step 2: Compute the barycenters fig: sf2: PDM2d

Step 3: Select the barycenter with the largest error fig: sf3: PDM2d

Step 4: Update the polytope division fig: sf4: PDM2d

Step 5: Compute the new barycenters fig: sf5: PDM2d

FIGURE 1. Depiction of PDM in 2-dimensional parameter case fig: PDM2d

Although PDM is a way forward to overcome the curse of dimensionality, the size of the reduced basis can still become prohibitively large if we want a sufficient approximation of the solution manifold. One approach to mitigate this challenge might be through hyperreduction techniques, which

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involve partitioning the parameter space into subspaces and determining a separate reduced basis for each subspace. The reduced bases in these subspaces are typically much smaller in size. Hyperreduction techniques have been applied successfully in combination with some reduced basis construction methods, but never in combination with the PDM.

In this project, we will explore the integration of the Polytope Division Method with hyperreduction techniques, aiming to improve computational efficiency and scalability in solving PDEs with high-dimensional parameter spaces.

2. Tasks

- Read and understand literature (provided by supervisor)
- Understand the Python script of PDM and learn to work with it
- Implement hyperreduction methods for examples
- Combine the hyperreduction method with PDM and compare this with original hyperreduction methods.

3. Requirements

- Programming experience with Python in general
- Some knowledge of Partial Differential Equations and Finite Elements

4. Supervisors

Karen Veroy - Grepl — k.p.veroy@tue.nl
Evie Nielen — e.t.e.nielen@tue.nl